SampleOutputTunisia.pdf

This file shows sample output of `seascorr` run on a standard tree-ring index from Tunisia and gridded monthly precipitation P and temperature T data. The data are the same used by (Meko et al., 2011). Relevant input settings are:

1. P as primary climate variable, T as secondary
2. September as the ending month of tree-ring growth
3. 1 month, 3 months, 9 months, 12 months as the four season-lengths
4. 1 000 as the number of simulations
5. Color output — rather than black and white
6. 1903-2002 as analysis period; 1903-52 and 1953-2002 as the early and late sub-periods

The next 11 pages show annotated `seascorr` output figure windows 1-11. That is followed by a mathematical description of the difference-of-correlation test of figure-window 11, and by screen captures illustrating the contents of output argument ”Result'.

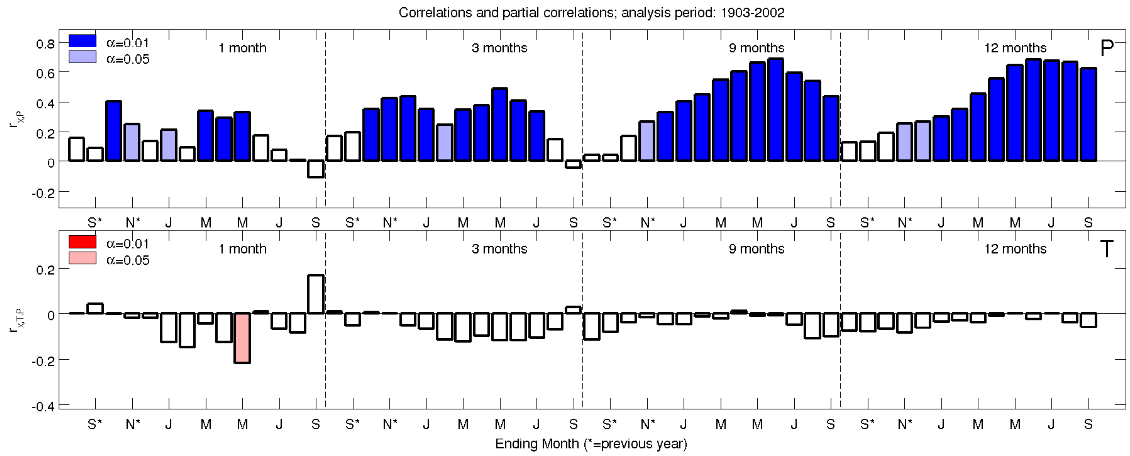


Figure 1: Correlations and partial correlations of tree-ring series with seasonalized climate variables. (Top) Simple correlations with the primary climate variable, P. (Bottom) Partial correlations of tree-ring index with secondary climate variable, T. Significance at $\alpha = 0.05$ and $\alpha = 0.01$ color-coded. Notation $r_{x,P}$ means correlation of x (tree-ring series) with P (precipitation); $r_{x,T.P}$ means partial correlation of x with T (temperature), controlling for influence of P.

Correlation of P with x is positive and significant for six months between October preceding the growth year and May of the Growth year. P correlation increases with summing over months. The 3-month sum reaches a maximum for Mar-May, the 9-month sum for Oct-June. Correlation does not increase further as July, Aug and Sept are brought into the sum. This plateau in response is reasonable, as July-September P is poorly correlated with x. T partial correlation is significant for May only. This May correlation is negative — consistent with drought stress of low precipitation exacerbated by high May temperature. September T partial correlation is relatively large and positive, but not significant. The T partial correlation becomes smaller with averaging over months because some individual months because there is no consistent large same-sign influence of T on x for a consecutive block of months.

Note this figure does not give the simple correlation of T with x. That could readily be checked, however, by running `seascorr` with the same settings and data except exchanging the roles of P and T as primary and secondary climate variables. The partial correlation of T with x — adjusting for P — may differ from the simple correlation whenever correlations of P with T and P with x are non-zero (Meko et al., 2011). The correlation of P with T can be checked with Figure 2 (below).

Because Monte Carlo sampling is random, a bar marked ”significant” in one run of `seascorr` might not be marked significant in a second run. This can happen especially if the sample correlation or partial correlation is near the threshold for empirical statistical significance. The significant negative T partial correlation in May is a good example of this sensitivity. The partial correlation for may is $r = 0.243$, which extremely close to the threshold for significance (0.025) at $\alpha = 0.05$. Repeated runs of `seascorr` sometimes indicate the sample $r = 0.243$ not significant at $\alpha = 0.05$.

The construction of this figure is described in more detail in Meko et al. (2011).

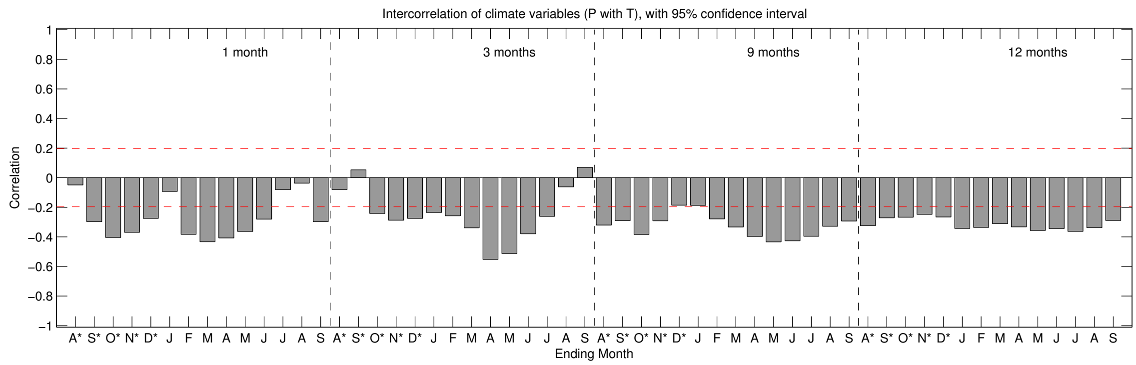


Figure 2: Seasonal correlations of primary with secondary climate variable (P with T). Correlations are plotted for each of 56 seasons defined by 14 different ending months and four season-length. Bars are proportional to simple correlations. Confidence interval is approximate, with no adjustment for serial correlation, non-normality or multiple comparisons. The approximate 95% confidence interval is set at $0 + 1.96/\sqrt{N}$, where N is the sample size.

Correlation of P with T is negative for all months of the year and significant for 10 months. Because the correlation is same-sign in individual months, the negative correlation carries over to multi-month seasons. The negative correlation of P with T can complicate interpretation of simple correlations of T with tree-ring index, x, if x is also correlated with P. The objective of the partial correlation analysis in `seascorr` is to help identify any significant "independent" relationship of T with x, where "independent" refers to independent of the influence of P.

Intercorrelation of the climate variables P and T is expected on physical grounds, depending on the climate regime, and might change sign from one season to another. For example, in semiarid continental regions, warm-season correlations of P and T are sometimes negative and cool-season correlations positive. The negative correlations are due to land-surface energy-balance relationships, with differing allotment of energy to latent and sensible heat. Relationships of incident radiation to precipitation through cloudiness can also be important. For example, wet summers may come with increased cloudiness, reduced solar radiation and surface heating, and energy spent evaporating water from the surface rather than in heating the surface and raising air temperature. Cool-season correlations could also reflect such factors, but also may reflect temperature advection patterns associated with synoptic precipitation-delivering storms. For example, in some continental locations wet winter storms may be associated with advection of warm, moist air from the south.

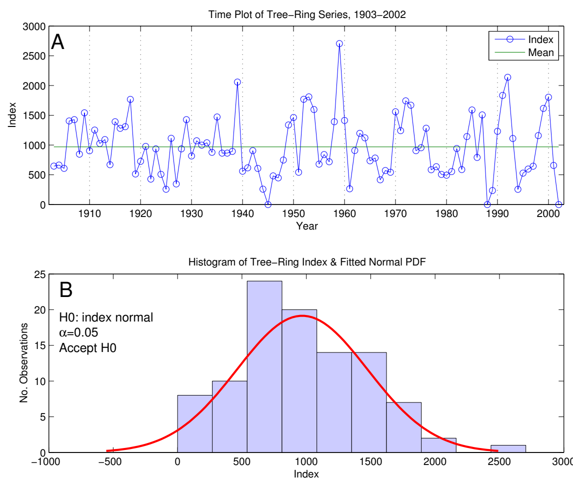


Figure 3: Time variation and frequency distribution of tree-ring index. (A) Time plot, 1903-2002. (B) Histogram and fitted normal probability density function. Results of Lilliefors test for normality (Conover, 1980) annotated at upper left of (B).

The tree-ring series varies greatly from year to year, and also has considerable variation from decade to decade. The series is approximately normally distributed. The plotted segment of tree-ring series is the only part of the tree-ring record used in any analysis (e.g., correlations, spectrum) by `seascorr`.

The time plot of the tree-ring series can draw attention to possible data errors, exceptional low-growth or high-growth years, and temporal changes in variance and other statistical properties. With `seascorr`, a check for normality is important because exact simulation is specifically for Gaussian series (Percival and Constantine, 2006). Seascorr uses the Lilliefors test (Conover, 1980) for normality. The histogram can be used for visual assessment of normality by comparison with the probability density function of normal variable with the same mean and variance as the tree-ring index.

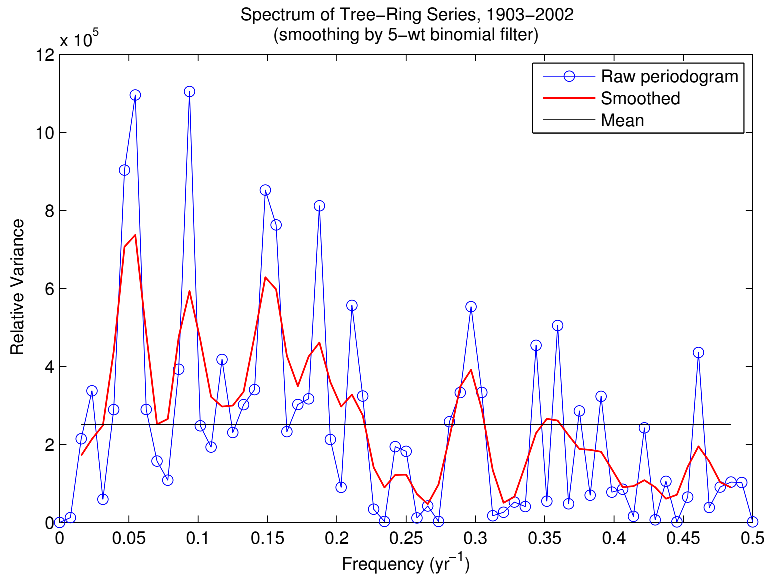


Figure 4: Spectrum of tree-ring index. The raw periodogram is the roughest possible estimate of the spectrum of the generating process of the series. Periodogram plotted has 65 points, which are at the zero frequency and the other Fourier frequencies corresponding to the padded series length of 128 (next power of 2 higher than the series length). Smoothing is by binomial weights 0.0625 0.2500 0.3750 0.2500 0.0625.

The smoothed-periodogram spectrum shows relatively high variance at frequencies 0.05-0.20, corresponding to wavelengths 20 yr to 5 yr. A somewhat low-frequency spectrum might be expected because the tree-ring index is ”standard” rather than ”residual” (Cook, 1985).

The spectrum of the tree-ring series, x, is central to estimation of confidence intervals of correlations and partial correlations in `seascorr` because the Monte Carlo simulations of x by exact simulation are generated such that they have the same as the observed x (Meko et al., 2011). The raw-periodogram values indeed are used directly in the equations for simulation. It should be noted that in the exact simulation algorithm of `seascorr` x is padded to the next power of 2 higher than 4 times the sample length of the original x, while in the periodogram above the series has been padded to the next power of 2 higher than 2 times the sample length of the original x.

The spectrum in `seascorr` is estimated using the same segment of tree-ring index as used for correlations with climate variables. Thus we would see a different spectrum for the full length of available tree-ring index (e.g., hundreds of years). The horizontal line at the mean of the 65 periodogram ordinates represents a theoretical white noise spectrum. No statistical test has been applied here to for statistical significance of departures of the observed spectrum from the white noise spectrum.

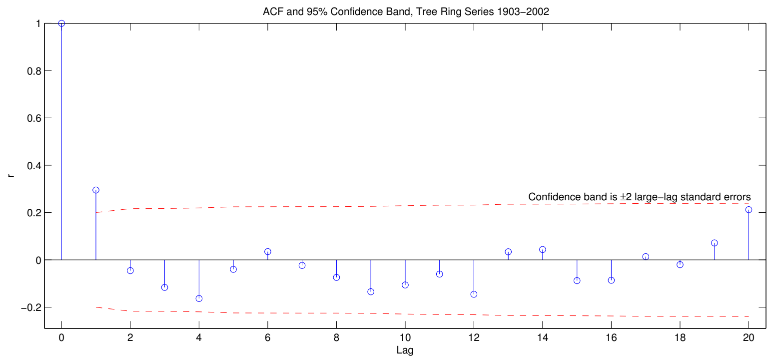


Figure 5: Sample autocorrelation function of tree-ring index. Confidence band from large-lag standard error (Box and Jenkins, 1976).

Only the lag-1 autocorrelation is statistically significant. The small lag-1 coefficient ($r_1 = 0.30$) corresponds to less than 10 percent of index variance explained by dependence on the previous year's index. Highly autocorrelated tree-ring series are problematic for `seascorr` if the corresponding seasonal climate data have little or no autocorrelation. This undesirable situation can be identified readily with `seascorr` through comparison of lag-1 autocorrelations of tree-ring climate data (See Figure 6). Two possible solutions for a miss-match of autocorrelation in tree-ring and climate series are to 1) remove that portion of the tree-ring index predictable from past-years' index by autregressive modeling or 2) use the residual rather than standard index (Cook, 1985).

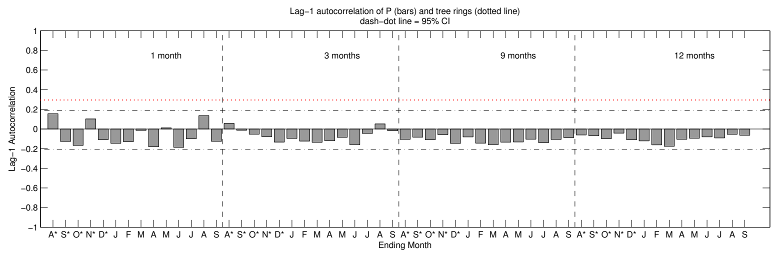


Figure 6: Lag-1 autocorrelations of tree-ring variable and primary climate variable. Dotted line shows lag-1 autocorrelation r_1 of the tree-ring series. Bars show r_1 of P for each of the 56 seasons. Dashed lines show approximate 95% confidence interval of lag-1 autocorrelation for a random series with the given sample length. With certain assumptions, this interval is appropriate for testing a two-sided null hypothesis of zero first-order autocorrelation (Haan, 2002).

The relevant comparison is the height of the bars (climate autocorrelation) with the distance of the dotted line from zero (lag-1 tree-ring autocorrelation). A large difference indicates a miss-match of autocorrelation in the tree-ring series and the climate series (see caption to Figure 5). The small but significant lag-1 autocorrelation of this tree-ring series contrasts with a generally low lag-1 autocorrelation of P (bars inside confidence interval). An argument could be made for using the residual tree-ring index for this analysis, but partly for illustration purposes in Meko et al. (2011) and partly because the tree-ring autocorrelation is small the standard index was used.

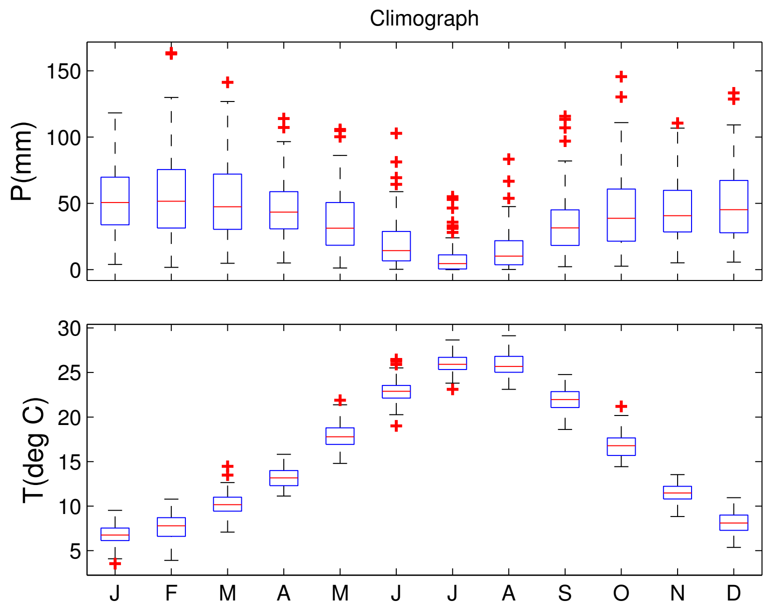


Figure 7: Climograph of monthly climate data. Box shows location of middle quartile of monthly observations. Horizontal line in middle of box is median. The whiskers extend to the most extreme data value not considered an outlier. An outlier is any data value more than 1.5*d* above top of box or more than 1.5*d* below the bottom of box, where *d* is the interquartile range. If no outliers, the whiskers are at the data extremes.

A climograph summarizes the broader aspects of climatology of the site, and can help in the interpretation of `seascorr` results by pointing out months in which the climate variable is more-or-less likely to be limiting to growth. For example, sensitivity may increase as growing-season temperature warms above some threshold level in spring, or may be amplified for months with a wide range of variability of precipitation from year to year. The *P* plot above is classic Mediterranean: precipitation highest in winter and very low in summer. From the outliers in the *P* plot, we see that although July is typically very dry, exceptionally wet July's can be wetter than the typical April. The *T* plot shows hottest and driest months are July and August. The outliers show that in no year is *P* zero for any month. This is partly an artifact of using gridded climate data. At individual stations, greater extremes from year to year are expected. Conditions represented by the gridded data will also differ from those experienced at the tree-ring site.

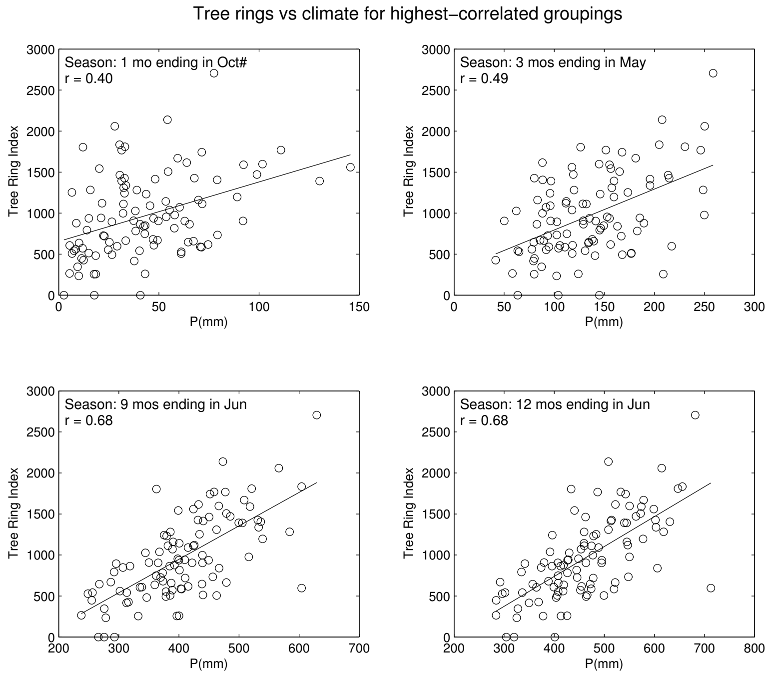


Figure 8: Scatterplots of tree-ring series on seasonalized primary climate variable for ”best’ seasons. These seasons are defined as the seasons with highest correlation for each of the four specified season-lengths. A least-squares-fit straight line and correlation coefficient are annotated on each plot.

The scatterplots are intended for quality control to check that correlations actually reflect general patterns of data and not outliers, and that relationships are indeed linear. Relationships in the plots above appear to be fairly linear, and not driven by outliers. A linear reconstruction model might therefore be appropriate for seasons with a strong signal — for example, annual precipitation summed over July-June. Note that the ”best” seasons correspond to the largest absolute correlations in the top bar-plot of Figure 1. Time series of tree-ring index and P for those seasons are plotted in Figure 9, and a summary table for the four seasons is shown in Figure 10.

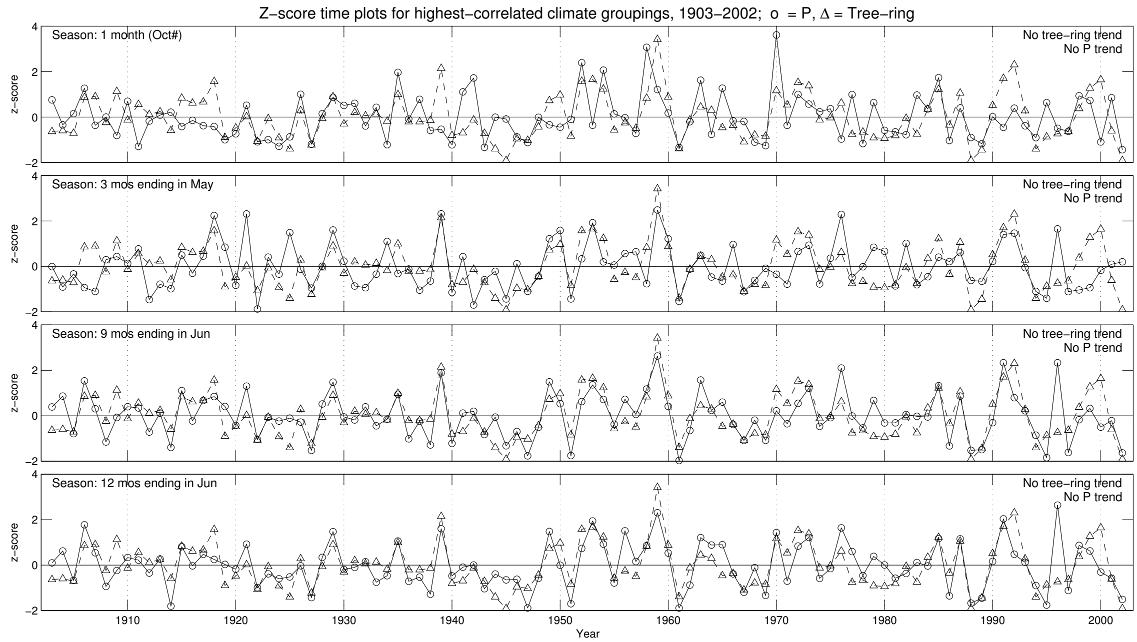


Figure 9: Time plots of tree-ring index and seasonalized primary climate variable for highest-correlated seasons. Series converted to "z-scores" (zero mean, unit variance) before plotting. Annotated at upper right are results of a *t*-test of the slopes of a regressions of the two series on time. Significance at 0.05 or 0.01 is flagged.

In general, the plots show that high growth goes with wetter conditions, and vice versa. The increase in ability of tree-ring index to track P with integration of P over several months is obvious in comparison of the top plot with lower plots. There are, however, notable exceptions. Bad misses, such as for annual P in 1996 (bottom) plot suggest a closer analysis of the climate data. For example, monthly or daily P could be examined for unusual sequence of weather conditions in 1996. None of the series have significant trend.

The data for these plots is the same as the data for the scatterplots in Figure 8. The time plots allow identification of years in which the agreement of tree-ring index and P was especially good or bad. A trend in one series and not another may indicate influence of some variable other than the primary climate variable for that season on the tree-ring index, or might indicate a data problem. If an artifact of data processing or a data error (e.g., improper detrending of tree-ring data, or station move in rain gage), trend could lead to misleading correlations (either too high or too low, depending on the direction of trends) between tree rings and P.

```
HIGHEST-CORRELATED SEASONS
 (Tree rings with P)

Analysis period: 1903-2002
1000 simulations

--------------------------------
       Ending      Nonexceedance
 m     Month    r    probability
--------------------------------
 1      Oct#   0.40     0.9990 **
 3      May    0.49     0.9990 **
 9      Jun    0.68     0.9990 **
 12     Jun    0.68     0.9990 **
--------------------------------
# after month means "previous year"
m-month seasons with given ending month
r is highest correlation for each m

For complete list of correlations and
partial correlations that are plotted
in Figure 1, type 'Result.S{i}' at
  command prompt, where i is 1, 2, 3 or 4.

LIST OF FIGURE WINDOWS
  1) bar charts -- tree ring vs climate
  2) bar chart -- P vs T
  3) time plot and histogram, tree ring
  4) spectrum of tree-ring series
  5) acf of tree-ring series
  6) lag-1 autocorrelation comparison
  7) climogram
  8) scatter plots, tree-ring vs P
  9) time plots of tree rings and P
 10) this summary text window
 11) difference-of-correlation test
     (early vs late sub-periods)
```

Figure 10: Summary figure window listing seasonal groupings of P most highly correlated with tree rings and describing contents of other figure windows. Significance of correlation is flagged at $\alpha = 0.05$ by one asterisk and at $\alpha = 0.01$ by two asterisks. The non-exceedance probability is the probability point of the listed correlation in the empirical cumulative distribution function of the simulation-based correlations. For example, a large positive correlation significant at $\alpha = 0.01$ would have a non-exceedance probability greater than 0.995, and a large negative correlation significant at $\alpha = 0.01$ would have a non-exceedance probability lower than 0.005 (two-tailed test)

This figure window is a "quick" reference summarizing the most important information from the bar plot at top of Figure 1. The correlations listed above correspond to the longest bars for each of the 4 season-lengths. Note that the non-exceedance probability is listed as 0.9990 for all 4 season-lengths. That is because the observed highest correlation for each season-length is higher than for any of the 1 000 corresponding simulation-based correlations. The Weibull formula used for non-exceedance probability with a sample size of 1000 (1000 simulations) cannot be higher than 0.9990 or lower than 0.001(see Meko et al. (2011).

TEMPORAL STABILITY OF CORRELATION FROM EARLY TO LATE SUB-PERIOD

Full = 1903-2002, Early = 1903-1952, Late = 1953-2002

Season[a]		Correlation[b]			Sample Size[c]		Test Results[d]	
Months	length	Full	Early	Late	N_1	N_2	ΔZ	p
Oct*	1	0.40	0.31	0.46	50	50	-0.1712	0.406
Mar-May	3	0.49	0.46	0.52	50	50	-0.0743	0.719
Oct*-Jun	9	0.68	0.67	0.69	50	50	-0.0311	0.880
Jul*-Jun	12	0.68	0.68	0.68	50	50	-0.0022	0.991

[a]Season: start & end months and number of months in season; asterisk denotes year preceding tree-ring year.

[b]Correlation: Pearson correlation of tree-ring index with primary climate variable for full-period, early-period, and late-period.

[c]Sample Size: N_1 and N_2 are the effective sample sizes for the correlations computed on early and late sub-periods, respectively. Effective sample size is fewer than the number of observations if both time series have positive lag-1 autocorrelation. Autocorrelations for the assessment computed on the full analysis period. Sample-size adjustment after Dawdy and Matalas (1964).

[d]Test Results: The test statistic (ΔZ) is the difference between transformed correlations for the early and late periods, following Panofsky and Brier (1968) and Snedecor and Cochran (1989). The last column is the p-value for a test of the null hypothesis that the population sample correlations for the early and late period are the same. A significant difference in sub-period correlations is indicated by a small p (e.g., $p<0.05$).

Figure 11: `Seascorr` figure-window summarizing test of difference of correlation of tree-ring index with primary climate variable in early and late sub-periods. Columns defined in footnote to table. The p-value in the last column is the probability of the sample correlations for late and early periods differing as much as observed when the population correlation coefficients (unknown) are equal. Significant difference of correlation in early and late periods at $\alpha = 0.05$ would be indicated by $p < 0.05$.

This test is intended as a crude assessment of possible temporal instability in the relationship between the tree-ring index and primary climate variable. If the period of overlap of tree rings with instrumental climate data is short, such instability may be difficult to identify with the existing data, as sampling variability alone will usually lead to some differences in correlation. Moreover, the climate record itself may not be homogeneous due to station moves and other factors, and the climate station is rarely if ever at the tree-ring site. The user should also be aware of assumptions in the difference-of-correlation test (see next page).

The results above show no evidence for temporal instability of correlations from the early to late sub-periods. Sample correlations are essentially the same for the first and last halves of the 1903-2002 analysis period. In fact, for the 12-month grouping they both round to $r = 0.68$. The p-values are nowhere near 0.05, indicating we cannot reject the null hypothesis that the true (unknown) population correlation is the same for the early and late periods.

Difference-of-Correlation Test: Mathematical Description

The test follows (Snedecor and Cochran, 1989). Consider two pairs of samples, (X_1, Y_1) and (X_2, Y_2). These are assumed to be independent random samples from bivariate normal distributions. In our application, (X_1, Y_1) and (X_2, Y_2) are a tree-ring index and climate time series from non-overlapping sub-periods of a ”full-analysis” period. These early and late parts of the record are assumed to be non-overlapping and to be of length n_1 and n_2 years, respectively. Let the population correlations(unknown) between X and Y in the early and late periods be ρ_1 and ρ_2, and the corresponding sample correlations be r_1 and r_2.

The test addresses the null hypothesis $\rho_1 = \rho_2$, or $\rho_1 - \rho_2 = 0$, and uses the sample correlations r_1 and r_2 along with sample sizes n_1 and n_2 to compute a test statistic, ΔZ. The two sample correlations are first transformed by the Fisher transformation

$$z = (1/2)[ln(1 + r) - ln(1 - r)].$$

It can be shown that z is approximately normally distributed with a standard error of $\sigma_z = 1/sqrt(n-3)$, where n is the sample size (Fisher, 1915). The Fisher-transformed sample correlations, z_1 and z_2, are therefore each approximately normally distributed with standard errors $\sigma_{z_1} = 1/sqrt(n_1 - 3)$ and $\sigma_{z_2} = 1/sqrt(n_2 - 3)$, respectively. The test statistic is the difference

$$\Delta Z = z_2 - z_1$$

of the transformed correlations, and is approximately normally distributed with standard error $\sigma_{z_1} + \sigma_{z_2}$ (Snedecor and Cochran, 1989).

We use the above equations for the difference-of-correlation test, with the exception that we use ”effective” sample sizes, as defined by Dawdy and Matalas (1964) instead of the original sample sizes n_1 and n_2 for computation of the standard errors of z_1 and z_2. Dawdy and Matalas (1964) show that when both time series, x and y, are positively autocorrelated the effective sample size for assessing the standard error of a correlation coefficient is approximately

$$\tilde{n} = n(1 - r_{1_x} r_{1_y})/(1 + r_{1_x} r_{1_y})$$

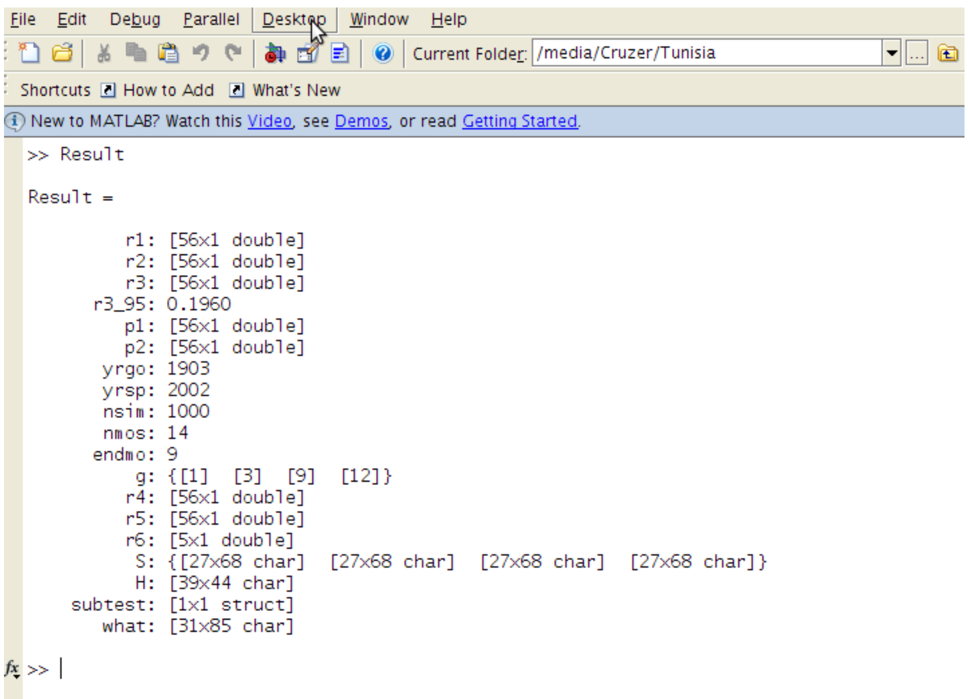
where r_{1_x} and r_{1_y} are the lag-1 autocorrelation coefficients of x and y, and n is the original sample size. This adjustment to effective sample size is made only if both time series have positive lag-1 autocorrelation. Otherwise, the original sample size is used. To obtain a robust estimate of autocorrelation applicable to the processes generating the time series x and y, the lag-1 autocorrelations are computed using the full period of `seascorr` analysis rather than the sub-periods.

Output argument "Result"

Whether called from a script or from the command window, `seascorr` returns as an output argument the structure variable "Result". The fields of this structure are described in detail in the opening comments section of `seascorr`. The output "Result" for the test data are provide in the file SampleOutputTunisa.mat. To view the data, load the file and type

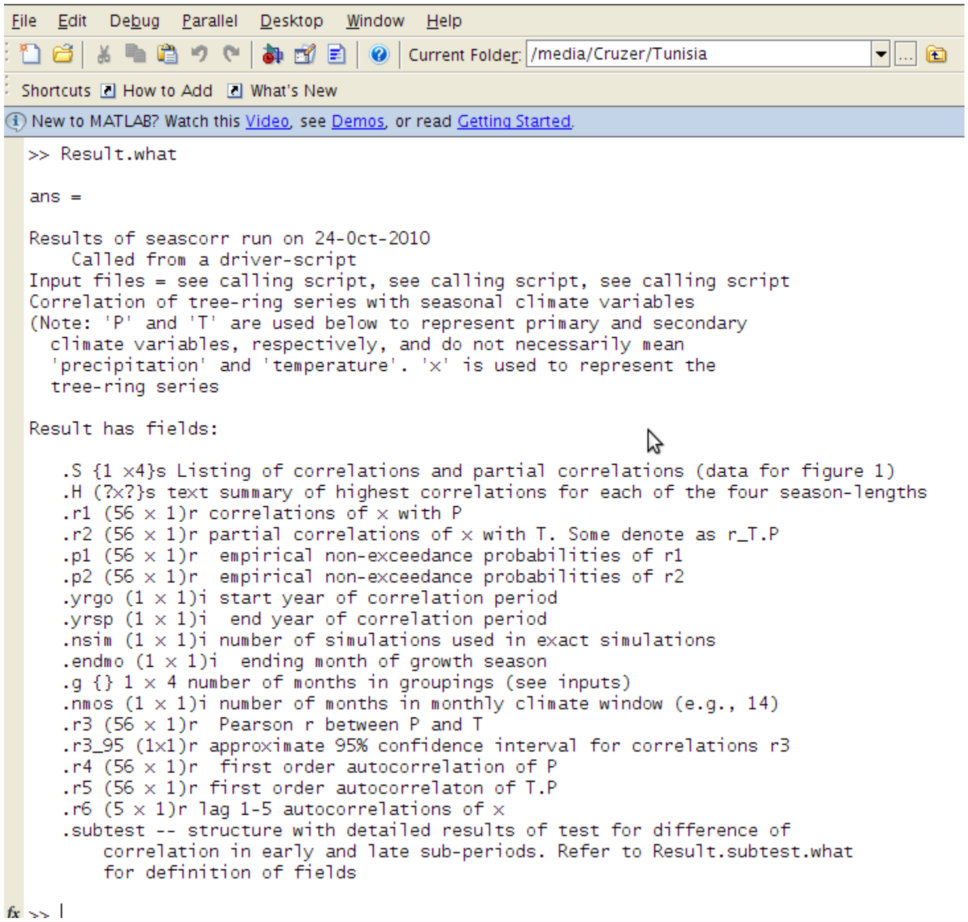
```
Result
```

at the MATLAB prompt to get the following (a screen capture):



The field what in structure Result describes each field in the structure. At the MATLAB prompt, type

```
Result.what
```

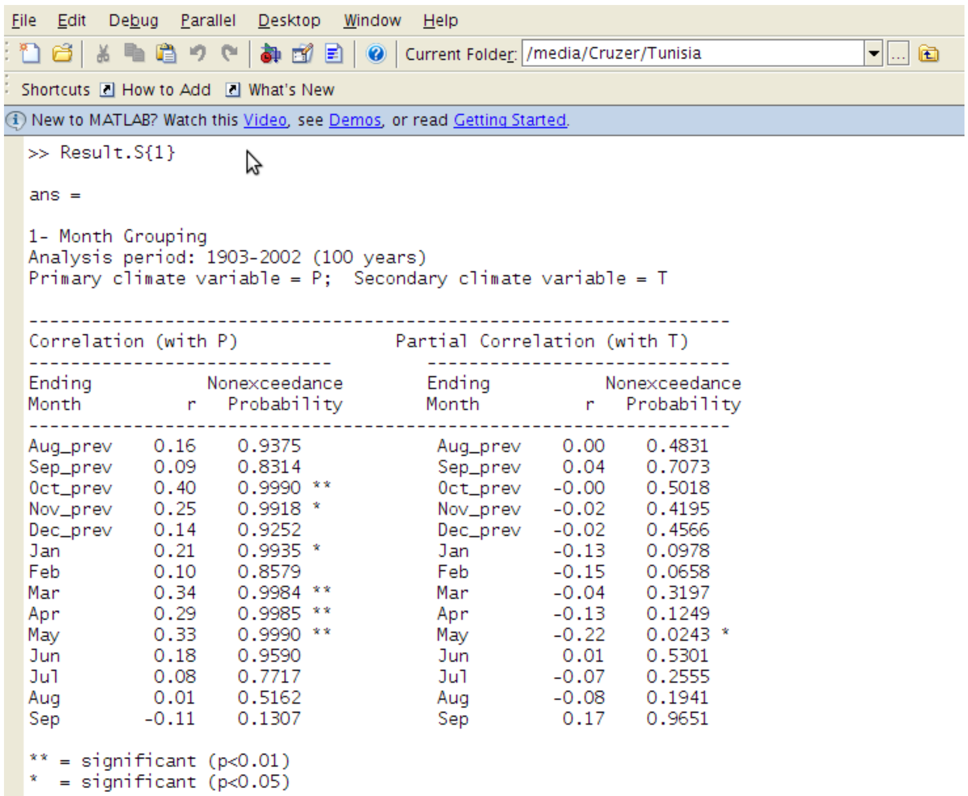



```
>> Result.what

ans =

Results of seascorr run on 24-Oct-2010
    Called from a driver-script
Input files = see calling script, see calling script, see calling script
Correlation of tree-ring series with seasonal climate variables
(Note: 'P' and 'T' are used below to represent primary and secondary
  climate variables, respectively, and do not necessarily mean
  'precipitation' and 'temperature'. 'x' is used to represent the
  tree-ring series

Result has fields:

   .S {1 x4}s Listing of correlations and partial correlations (data for figure 1)
   .H (?x?}s text summary of highest correlations for each of the four season-lengths
   .r1 (56 x 1)r correlations of x with P
   .r2 (56 x 1)r partial correlations of x with T. Some denote as r_T.P
   .p1 (56 x 1)r  empirical non-exceedance probabilities of r1
   .p2 (56 x 1)r  empirical non-exceedance probabilities of r2
   .yrgo (1 x 1)i start year of correlation period
   .yrsp (1 x 1)i  end year of correlation period
   .nsim (1 x 1)i number of simulations used in exact simulations
   .endmo (1 x 1)i  ending month of growth season
   .g {} 1 x 4 number of months in groupings (see inputs)
   .nmos (1 x 1)i number of months in monthly climate window (e.g., 14)
   .r3 (56 x 1)r  Pearson r between P and T
   .r3_95 (1x1)r approximate 95% confidence interval for correlations r3
   .r4 (56 x 1)r  first order autocorrelation of P
   .r5 (56 x 1)r first order autocorrelaton of T.P
   .r6 (5 x 1)r lag 1-5 autocorrelations of x
   .subtest -- structure with detailed results of test for difference of
       correlation in early and late sub-periods. Refer to Result.subtest.what
       for definition of fields

fx >> |
```

The above list indicates that correlations and partial correlations in Figure 1 of seascorr can be obtained from the fields Result.S{1}, Result.S{2}, Result.S{3} and Result.S{4}. Those four fields hold results for the 1-month, 3-month, 9-month, and 12-month seasons for the sample data.

A table for the 1-month seasons, for example, can be obtained with

```
Result.S{1}
```




```
>> Result.S{1}

ans =

1- Month Grouping
Analysis period: 1903-2002 (100 years)
Primary climate variable = P;  Secondary climate variable = T

-------------------------------------------------------------------
Correlation (with P)                 Partial Correlation (with T)
-----------------------------        -----------------------------
Ending             Nonexceedance     Ending             Nonexceedance
Month          r    Probability      Month          r    Probability
-------------------------------------------------------------------
Aug_prev    0.16    0.9375            Aug_prev    0.00    0.4831
Sep_prev    0.09    0.8314            Sep_prev    0.04    0.7073
Oct_prev    0.40    0.9990 **         Oct_prev   -0.00    0.5018
Nov_prev    0.25    0.9918 *          Nov_prev   -0.02    0.4195
Dec_prev    0.14    0.9252            Dec_prev   -0.02    0.4566
Jan         0.21    0.9935 *          Jan        -0.13    0.0978
Feb         0.10    0.8579            Feb        -0.15    0.0658
Mar         0.34    0.9984 **         Mar        -0.04    0.3197
Apr         0.29    0.9985 **         Apr        -0.13    0.1249
May         0.33    0.9990 **         May        -0.22    0.0243 *
Jun         0.18    0.9590            Jun         0.01    0.5301
Jul         0.08    0.7717            Jul        -0.07    0.2555
Aug         0.01    0.5162            Aug        -0.08    0.1941
Sep        -0.11    0.1307            Sep         0.17    0.9651

** = significant (p<0.01)
*  = significant (p<0.05)
```

This table lists the correlations and partial correlations plotted in Figure 1 in a form that can be cut and pasted into reports. Similarly, other statistical and tabular output can be obtained from the structure `Result`. Refer to the comment section of `seascorr` for definitions and description of the available output.

References

Box, G. E. P., Jenkins, G. M., 1976. Time Series Analysis: Forecasting and Control. Holden Day, San Francisco, 576 pp.

Conover, W., 1980. Practical Nonparametric Statistics, 2nd Edition. John Wiley & Sons, New York, 493 pp.

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